

MAC-CPTM Situations Project

Situation 16: Area of Sectors of a Circle

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Center for Proficiency in Teaching Mathematics
Revised 27 September 2005 – Dennis Hembree

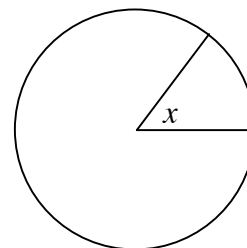
Edited at the University of Georgia
Center for Proficiency in Teaching Mathematics
07 December 2008 – Sharon K. O’Kelley and Shawn Broderick

Prompt

An honors geometry class had the following problem for homework. A student asked simply, “How do you do this?”

Complete the following table for a circle with radius 3:

Central Angle x	30°	60°	90°	120°	150°
Area of sector					



Question: Perhaps using a spreadsheet or writing a simple program for a hand-held computer such as the TI-83 best approaches this problem. What could this problem afford beyond a series of simple calculations?

Commentary

This prompt allows for the introduction of radian measure as a more efficient representation of the sector’s area formula. Students can also explore possible values for the area using both the table and programming features of a graphing calculator. In addition, software applications such as Excel are also useful in calculating the area. Such uses of technology allow the students to find any value within the domain and range of the continuous function represented by the formula.

Mathematical Foci

Mathematical Focus 1

The formula for the area of a sector can use either degree or radian measure.

The formula for the area of a sector, $A_x = \frac{x}{360}(\pi r^2)$, is straightforward, though cumbersome. Converting the central degree measure x , to radians θ , gives $A_\theta = \frac{\theta}{2}r^2$. This formula is less cumbersome and has the same form as the familiar area formula for a circle. The use of radian measure would require the students not only to use a modified version of the original formula but it would also require that they convert the central angles noted in the chart to radian measure as well. In most American geometry courses, students have not seen radian measure and this is an opportunity for entry into the topic.

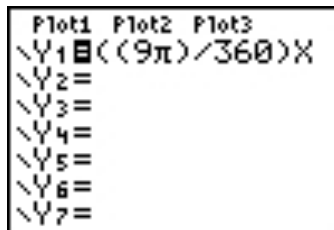
Mathematical Focus 2

Using a graphing calculator such as the TI-84, students can use the graphing and table features of the calculator to find the area of any sector.

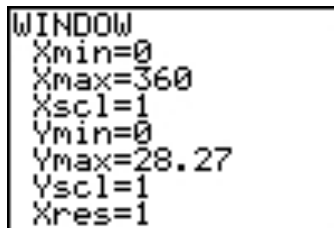
Given that we are discussing a circle with a radius of 3, the formula for the area of the sector becomes

$$A_x = \frac{x}{360}(\pi 3^2)$$

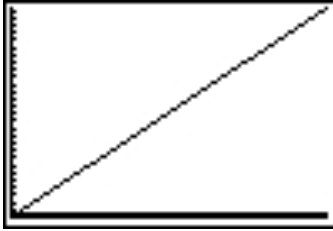
where x is the measure of the central angle. Plotting the points (x, A_x) on the Cartesian plane suggests there is a linear relationship between x and A_x . The formula or linear function, therefore, can be entered into the calculator using the **Y=** menu. The screen should look like the one below....



Upon analysis, questions of domain and range should arise. Since the angles of a circle vary from 0 degrees to 360 degrees, the domain of the linear function is $0 \leq x \leq 360$. In turn the range will vary from 0 to 9π or approximately 28.27 units. Knowing the domain and range allows the students to set the windows on their calculators.



These settings will produce the following graph....



To complete the given chart, students can use the **TBLSET** menu and set the table to 30-degree increments.

TABLE SETUP	
TblStart=0	■
ΔTbl=30	
Indent: Auto	Ask
Depend: Auto	Ask

Next, the students can access the **TABLE** menu to complete the chart. In addition, they can use the scroll feature of the calculator to find values greater than the 14.137 given below.

X	Y1	
0	0	
30	2.3562	
60	4.7124	
90	7.0686	
120	9.4248	
150	11.781	
180	14.137	
X=0		

It is worth noting that the **TBLSET** menu can be adjusted so that students can determine any value within the range of the continuous function. This requires that the independent variable of x to be set to **Ask**.

TABLE SETUP	
TblStart=■	
ΔTbl=30	
Indent: Auto	Ask
Depend: Auto	Ask

Now, students under the **TABLE** menu can enter any value for x within the domain. Pressing **[ENTER]** will yield the corresponding value for y .

X	Y1	
83.6	6.5659	
104.2	8.1838	
X=		

Mathematical Focus 3

Using a program in the TI-84.

Here is a simple TI-84 program that automatically calculates the area of a sector given the central angle (in degrees) and the radius. This program will be able to calculate the areas of the sectors in the original question and be a useful tool for calculating the area of any sector of a circle the student may encounter in the future.

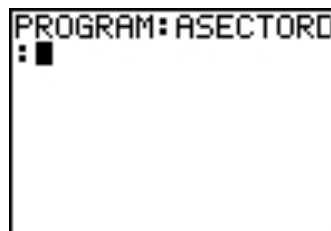
A program in the calculator is a set of commands the calculator executes sequentially, as though you had entered them in from the keyboard

This program is called **ASECTOR**, because it accepts the central angle of the sector (in degrees) and the radius as inputs and outputs the area of the sector, whether the calculator itself is in degree or radian mode.

1. Press **[PRGM]** **[→]** **[→]** to display the **PRGM NEW** menu.



2. Press **[ENTER]** to select **1:Create New**. The **Name=** prompt is displayed, and alpha-lock is on. Type in **[A][S][E][C][T][O][R][D]** as the name of your program.



You are now in the program editor. The colon (:) in the first column of the second line indicates the beginning of a command line.

3. Press **[PRGM] [→] [2]** to select **2:Prompt** from the **PRGM I/O** menu. **Prompt** is copied to the command line. Press **[ALPHA] [C]** as the name for the central angle variable (in degrees) of the sector. Press **[,]**. Press **[ALPHA] [R]** as the name for the radius variable of the circle. Press **[ENTER]**.

```
PROGRAM:ASECTORD
:Prompt C,R
:█
```

4. Press **[ALPHA] [C] [2nd] [π] [ALPHA] [R] [x²] [+][3][6][0][STO→][ALPHA] [A]** to enter the expression $\frac{x}{360}(\pi r^2)$ and store it to the variable **A**, for area (of the sector). Press **[ENTER]**.

```
PROGRAM:ASECTORD
:Prompt C,R
:CπR2/360→A
:█
```

5. Press **[PRGM] [→] [3]** to select **3:Disp** from the **PRGM I/O** menu. **Disp** is pasted to the command line. Press **[2nd] [A-Lock] ["] [A] [R] [E] [A] [] [O] [F] [] [S] [E] [C] [T] [O] [R] [:] ["] [ALPHA] [,] [ALPHA] [A] [,] [2nd] [A-Lock] ["] [] [U] [N] [I] [T] [S] ["]** to set up the program to display the text **AREA OF SECTOR:**, the calculated value of **A**, and **UNITS**.

```
PROGRAM:ASECTORD
:Prompt C,R
:CπR2/360→A
:Disp "AREA OF S
ECTOR:",A,"UNITS
"
:█
```

6. Press **[2nd] [Quit]** to display the home screen.

Now the program can be executed for the first central angle measure given in the table.

7. Press **[PRGM]** to display the **PRGM EXEC** menu. The items on this menu are the names of stored programs.

```
EXEC EDIT NEW
1 ASECTORD
```

8. Press **[ENTER]** to paste **prgmASECTORD** to the current cursor location. (If **ASECTORD** is not item **1** on your **PRGM EXEC** menu, move the cursor to **ASECTORD** before you press **[ENTER]**.)

```
Pr-gmASECTORD█
```

9. Press **[ENTER]** to execute the program. Enter **[3][0]** for the first “Central Angle x ,” and then press **[ENTER]**. Enter **[3]** for the radius, and then press **[ENTER]**. The text **AREA OF SECTOR IS:**, the value of **A**, **UNITS**, and **Done** are displayed.

```

PrgmASECTORD
C=?30
R=?3
AREA OF SECTOR:
UNITS 2.35619449
Done

```

Repeat steps 7 through 9 and enter the other values of “Central Angle x ” given in the table to fill in the rest of the “Area of sector” row.

You can also modify the current program lines to make a new program, **ASECTORR**, in order to calculate the area of a sector using radians, in radian mode.

```

PROGRAM:ASECTORR
:Promet C,R
:CR2/2→A
:Disp "AREA OF S
ECTOR:",A,"UNITS
"
:

```

1. Follow steps 1 and 2 from above to create a new program **ASECTORR**.
2. To copy all the command lines from **ASECTORD** to **ASECTORR**, do the following:
 - a. Press **[2nd][RCL]**. **Rc1** is displayed on the bottom line of the program editor in the new program.
 - b. Press **[PRGM][←]** to display the **PRGM EXEC** menu.
 - c. Select **ASECTORD** from the menu. **ASECTORD** is then pasted to the bottom line.
 - d. Press **[ENTER]**. All command lines from **ASECTORD** are now in **ASECTORR**.

3. Change the calculation line:
[C][π][R][x²][+][3][6][0][STO→][A] to
[C][R][x²][+][2][STO→][A].

3. Press **[2nd][QUIT]** to display the home screen. Press **[PRGM]** to display the **PRGM EXEC** menu. Execute **ASECTORR**. Enter **[2nd][π][+][6]** for the “Central Angle x ,” and then press **[ENTER]**. Enter **[3]** for the radius, and then press **[ENTER]**. The text **AREA OF SECTOR IS:**, the value of **A**, **UNITS**, and **Done** are displayed.

```

PrgmASECTORR
C=?π/6
R=?3
AREA OF SECTOR:
UNITS 2.35619449
Done

```

Mathematical Focus 4

Using a spreadsheet like Excel.

The table given in the prompt can be filled out using a spreadsheet and capitalizing on the ability to repeat operations.

1. Enter the data from the table into three columns of the spreadsheet, one for the radius

of the circle, the central angle, and area of sector:

	A	B	C
1	Radius of Circle	Central Angle x	Area of Sector
2	3	30	
3	3	60	
4	3	90	
5	3	120	
6	3	150	
7			

2. In cell C2, type in the equation:

	A	B	C
1	Radius of Circle	Central Angle x	Area of Sector
2	3	30	=B2*PI()*A2/360
3	3	60	
4	3	90	
5	3	120	
6	3	150	
7			

3. Press Enter and the solution is found instantly:

	A	B	C
1	Radius of Circle	Central Angle x	Area of Sector
2	3	30	2.35619449
3	3	60	
4	3	90	
5	3	120	
6	3	150	
7			

4. To fill in the rest of the table, click on cell C2, point to the small box on the lower right corner, and drag it down to cell C6. The solutions will be calculated using the numbers from columns A and B in the same row as column C:

	A	B	C
1	Radius of Circle	Central Angle x	Area of Sector
2	3	30	2.35619449
3	3	60	4.71238898
4	3	90	7.068583471
5	3	120	9.424777961
6	3	150	11.78097245
7			

Now our table is filled.

We can change all calculations to **radians** by the following five steps.

1. Insert a column in between column B and C by clicking on column C and going to Insert → Columns:

	A	B	C	D
1	Radius of Circle	Central Angle x		Area of Sector
2	3	30		2.35619449
3	3	60		4.71238898
4	3	90		7.068583471
5	3	120		9.424777961
6	3	150		11.78097245
7				

2. In cell C1, type in “Central Angle x (in radians).” Then, in cell C2, enter the equation:

	A	B	C	D
1	Radius of Circle	Central Angle x	Central Angle x (in radians)	Area of Sector
2	3	30	=B2*PI()/180	2.35619449
3	3	60		4.71238898
4	3	90		7.068583471
5	3	120		9.424777961
6	3	150		11.78097245
7				

3. Fill in the rest of the column:

	A	B	C	D
1	Radius of Circle	Central Angle x	Central Angle x (in radians)	Area of Sector
2	3	30	0.523598776	2.35619449
3	3	60	1.047197551	4.71238898
4	3	90	1.570796327	7.068583471
5	3	120	2.094395102	9.424777961
6	3	150	2.617993878	11.78097245
7				
8				

4. Now, enter in the new equation in for the “Area of Sector” column:

	A	B	C	D
1	Radius of Circle	Central Angle x	Central Angle x (in radians)	Area of Sector
2	3	30	0.523598776	=C2*A2^2/2
3	3	60	1.047197551	4.71238898
4	3	90	1.570796327	7.068583471
5	3	120	2.094395102	9.424777961
6	3	150	2.617993878	11.78097245
7				

5. Finally, overwrite the rest of the answers in degrees with the answers in radians. Notice that the answers did not change.

With this spreadsheet, we can change the radius of the circle and the central angle measures and then be able to calculate the area of any sector of the circle.